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Economic Theory

Module 3

BUS 802 Economic Theory Module 3

Course Developer/Writer
Dr. J.O. Onyemaechi, National Open University of Nigeria

Course Editor
Dr. A. O. Fagbemi, National Open University of Nigeria

Course Coordinator
Dr. O. Adenuga, National Open University of Nigeria

Programme Leader
Dr. C. I. Okeke, National Open University of Nigeria

Credits of cover-photo: Henry Ude, National Open University of Nigeria

National Open University of Nigeria - 91, Cadastral Zone, Nnamdi Azikiwe Express Way, Jabi, Abuja, Nigeria

www.nou.edu.ng centralinfo@nou.edu.ng
oer.nou.edu.ng oerunit@noun.edu.ng OER repository

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Module 3

Unit 1 The Theory of Production

1.0 Introduction

No matter the objective of any business organisation, achievement of efficiency in production or cost minimisation for a given production activity appear to be one of the prime concern of the managers. As a matter of fact, the survival of a business firm in a competitive environment depends on its ability to produce at competitive costs. Firms are, therefore, mandated to either minimise costs of production or maximise output from a given quantity of inputs. In the manager's effort to minimise production costs, the fundamental questions he or she faces are:

(a) How can production be optimised or costs minimised?

(b) What will be the beaviour of output as inputs increase?

(c) How does technology help in reducing production costs?

(d) How can the least-cost combination of inputs be achieved?

(e) Given the technology, what happens to the rate of return when more plants are added to the firm?

The theory of production attempts to provide theoretical answers to these questions, through abstract models built under hypothetical conditions. It follows that, though production theories may not provide solutions to the real life business problems, it can provide tools and techniques for the analysis of production conditions and for finding solutions to the practical business problems.

In this unit, we present the theory of production, using two input combinations, labour and capital.

2.0 Objectives

At the end of this unit, you should be able to:

- understand the basic principles of production
- know how to develop a production function for given input combinations
- begin to think in terms of optimal input combinations for your production activities.

3.0 Main Content

3.1 The Theory of Production

Production theory generally deals with quantitative relationships, that is, technical and technological relationships between inputs, especially labour and capital, and between inputs and outputs.

An ***input*** is a good or service that goes into the production process. As economists refer to it, an input is simply anything which a firm buys for use in its production process. An ***output***, on the other hand, is any good or service that comes out of a production process.

Economists classified inputs as (i) labour; (ii) capital; (iii) land; (iv) raw materials; and, (v) time. These variables are measured per unit of time and hence referred to as flow variables. In recent times, entrepreneurship has been added as part of the production inputs, though this can be measured by the managerial expertise and the ability to make things happen.

Inputs are classified as either ***fixed*** or ***variable*** inputs. Fixed and variable inputs are defined in both economic sense and technical sense. In *economic sense,* a fixed input is one whose supply is inelastic in the short run. In *technical sense*, a fixed input is one that remains fixed (or constant) for certain level of output.

A variable input is one whose supply in the short run is elastic, example, labour, raw materials, and the like. Users of such inputs can employ a larger quantity in the short run. Technically, a variable input is one that changes with changes in output. In the long run, all inputs are variable.

3.2 The Production Function

Production function is a tool of analysis used in explaining the input-output relationship. It describes the technical relationship between inputs and output in physical terms. In its general form, it holds that production of a given commodity depends on certain specific inputs. In its specific form, it presents the quantitative relationships between inputs and outputs. A production function may take the form of a schedule, a graph line or a curve, an algebraic equation or a mathematical model. The production function represents the technology of a firm.

An empirical production function is generally so complex to include a wide range of inputs: land, labour, capital, raw materials, time, and technology. These variables form the independent variables in a firm's actual production function. A firm's long-run production function is of the form:

$$Q = f(L_d, L, K, M, T, t) \qquad (3.2.1)$$

where L_d = land and building; L = labour; K = capital; M = materials; T = technology; and, t = time.

For sake of convenience, economists have reduced the number of variables used in a production function to only two: capital (K) and labour (L). Therefore, in the analysis of input-output relations, the production function is expressed as:

$$Q = f(K, L) \qquad (3.2.2)$$

Equation (3.2.2) represents the algebraic or mathematical form of the production function. It is this form of production function which is most commonly used in production analysis.

As implied by the production function (equation (3.2.2), increasing production, Q, will require K and L, and whether the firm can increase both K and L or only L will depend on the time period it takes into account for increasing production, that is, whether the firm is thinking in terms of the *short run* or in terms of the *long run.*

Economists believe that the supply of capital (K) is *inelastic* in the short run and *elastic* in the long run. Thus, in the short run firms can increase production only by increasing labour, since the supply of capital is fixed in the short run. In the long run, the firm can employ more of both capital and labour, as the supply of capital becomes elastic over time. In effect, there exist two types of production functions:

1. The short-run production function; and,

2. The long-run production function

The short-run production function often referred to as the ***single variable production function***, can be written as:

$$Q = f(L) \qquad (3.2.3)$$

In The long-run, both capital (K) and labour (L) is included in the production function, so that the long-run production function can be written as:

$$Q = f(K, L) \qquad (3.2.4)$$

A production function ***is based on the following assumptions***:

(i) perfect divisibility of both inputs and output;

(ii) there are only two factors of production – capital (K) and lacour (L);

(iii) limited substitution of one factor for the other;

(iv) a given technology; and,

(v) inelastic supply of fixed factors in the short-run.

Any changes in the above assumptions would require modifications in the production function.

The two most important forms of production functions used in economic literature in analysing input-output relationships are the *Cobb-Douglas* production function and the *Constant Elasticity of Substitution (CES)* production function. Our interest at this level will be limited to the Cobb-Douglas production function.

3.2.1 The Cobb-Douglas Production Function

The Cobb-Douglas production function is of the following general form:

$$Q = AK^aL^b \qquad (3.2.5)$$

where a and b are positive fractions.

The Cobb-Douglas production function is often used in its following form:

$$Q = AK^aL^{(1-a)} \qquad (3.2.6)$$

3.2.2 Properties of the Cobb-Douglas Production Function

A power function of the Cobb-Douglas type has the following important properties:

First, the multiplicative for of the power function (3.2.5) can be transformed into its log-linear form as:

$$\log Q = \log A + a \log K + b \log L \qquad (3.2.7)$$

In its logarithmic form, the function becomes simple to handle and can be empirically estimated using linear regression techniques.

Second, power functions are homogeneous and the degree of homogeneity is given by the sum of the exponents a and b as in the Cobb-Douglas function. If a + b = 1, then the production function is homogeneous of degree 1 and implies constant returns to scale.

Third, a and b represent the elasticity coefficient of output for inputs, K and L, respectively. The output elasticity coefficient (ε) in respect of capital can be defined as proportional change in output as a result of a given change in K, keeping L constant. Thus,

$$\varepsilon_k = \frac{\partial Q/Q}{\partial K/K} = \frac{\partial Q}{\partial K}.\frac{K}{Q} \qquad (3.2.8)$$

By differentiating the production function, Q = AKaLb, with respect to K and substituting the result into equation (3.2.8), the elasticity coefficient, ε_k, can be derived:

$$\frac{\partial Q}{\partial K} = aAK^{(a-1)}L^b$$

Substituting the values for Q (equation (3.2.5)) and ∂Q/∂K into equation (3.2.8), you get:

$$\varepsilon_k = a\,AK^{(a-1)}L^b \left[\frac{K}{AK^aL^b}\right]$$

$$= a$$

It follows that the output coefficient for capital, K, is ‘a’. The same procedure may be applied to show that ‘b’ is the elasticity coefficient of output for labour, L.

Fourth, the constants a and b represent the relative distributive share of inputs K and L in the total output, Q. The share of K in Q is given by:

$$\frac{\partial Q}{\partial K} . K$$

Similarly, the share of L in Q can be obtained by:

$$\frac{\partial Q}{\partial L} . L$$

The relative share of K in Q can be obtained as:

$$\frac{\partial Q}{\partial K} . K. \frac{1}{Q} = a$$

and the relative share of L in Q can be obtained as:

$$\frac{\partial Q}{\partial L} . L . \frac{1}{Q} = b$$

Finally, the Cobb-Douglas production function in its general form,

$Q = K^a L^{(1-a)}$, implies that at zero cost, there will be zero production.

Some of the necessary concepts in production analysis can be easily derived from the Cobb-Douglas production function as shown below:

1. Average Products of L (AP_L) and K (AP_K):

$$AP_L = A (K/L)^{(1-a)}$$

$$AP_K = A (L/K)^1$$

2. Marginal Products of L (MP_L) and K (MP_K):

$$MP_L = a(Q/L)$$

$$MP_K = (1 - a)Q/K$$

3. Marginal Rate of Technical Substitution of L for K ($MRTS_{L,K}$):

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{a}{(1\text{-}a)} . \frac{K}{L}$$

Note the MRTS $_{L,K}$ is the rate at which a marginal unit of labour, L, can be substituted for a marginal unit of capital, K (along a given isoquant) without affecting the total output.

3.2.3 Degree of Production Functions and Returns to Scale

The famous laws of returns to scale can be explained through production functions. Assume generally productions function involving two variables capital (K) and labour (L), and one commodity, X. The production function may be expressed in the form:

$Q_x = f(K, L)$ (3.2.9)

Q_x denotes the quantity produced of commodity X. Assume also that the production function is ***homogeneous***, that is, when all inputs are increased in the same proportion, the proportion can be factored out mathematically. If when all inputs are increased by a certain proportion (say, k) and output increases by the same proportion (k), the production function is said to be homogeneous of degree 1. A production function of homogeneous of degree 1 is expressed as follows:

$kQ_x = f(kK, kL)$ (3.2.10)

$= k(K, L)$

A homogeneous production function of degree 1 implies ***constant returns to scale***. Equation (3.2.10) indicates that increases in the inputs K and L by a multiple of k, will increase output, Q_x, by the same multiple, k, implying constant returns to scale.

Note that increasing inputs, say K and L in the same proportion may result in increasing or diminishing returns to scale. Simply stated, it is likely that increases in all the inputs in certain proportion may not result in increase in output in the same proportion. If all the inputs are doubled, for example, output may not be doubled; it may increase by less than or more than double. In this case, the production function can be expressed as:

$hQ_x = f(kK, kL)$ (3.2.11)

where h denotes h-times increase in output, Q_x, as a result of k-times increase in inputs, K and L. The proportion, h may be greater than k, equal to k, or less than k. This touches on ***the three laws of returns to scale***:

(i) f $h = k$, production function reveals constant returns to scale

(ii) If $h > k$, production function reveals increasing returns to scale

(iii) If $h < k$, the production function reveals decreasing returns to scale.

Observe that in the production function, equation (3.2.10), k has an exponent equal to 1 (that is, $k = k^1$), hence, it is of homogeneous of degree 1. In general, the exponent of k can take the letter r, where $r \neq 1$. A production function is therefore, said to be homogeneous of degree r when if all the inputs are multiplied by k, output increases by a multiple of K^r. That is, if,

$f(kK, kL) = K^r(K, L) = k^rQ$ (3.2.12), then the production function (equation, 3.2.12) is homogeneous of degree r.

From this production function, the laws of returns to scale can again be derived as follows:

(i) If $k > 1$, and $r < 1$, production function reveals decreasing returns to scale

(ii) If $k > 1$, and $r > 1$, production function reveals increasing returns to scale

(iii) If $k > 1$, and $r = 1$, production function reveals constant returns to scale.

Consider the following multiplicative form of a production function:

$$Q = K^{0.25}L^{0.50} \qquad (3.2.13)$$

If K and L are multiplied by k, and output increases by a multiple of h, then

$$hQ = (kK)^{0.25}(kL)^{0.50}.$$

factoring out k, you get:

$$hQ = k^{0.25 + 0.50}[K^{0.25}L^{0.50}]$$

$$= k^{0.75}[K^{0.25}L^{0.50}] \qquad (3.2.14)$$

According to equation (3.2.14), $h = k^{0.75}$ and $r = 0.75$, implying that $r < 1$, and, $h < k$. It follows that the production function (equation, 3.2.13) shows decreasing returns to scale.

Consider another production function of the form:

$$Q = f(K, L, X) = K^{0.75}\ L^{1.25}\ X^{0.50} \qquad (3.2.15)$$

Multiplying K, L, and X by k, Q increases by a multiple of h:

$$hQ = (kK)^{0.75}\ (kL)^{1.25}\ (kX)^{0.50}$$

Again factoring out k, you get:

$$hQ = k^{(0.75+1.25+0.50)}[K^{0.75}\ L^{1.25}\ X^{0.50}]$$

$$= k^{2.5}[K^{0.75}\ L^{1.25}\ X^{0.50}]$$

Observe that in this case, h = k2.5 and r = 2.5, so that $h > k$. Thus, production function (equation, 3.2.15) depicts increasing returns to scale.

Self-Assessment Exercise

Define production function and describe the underlying assumptions

4.0 Conclusion

The theory of production attempts to provide theoretical answers to these questions, through abstract models built under hypothetical conditions. Production function is a tool of analysis used in explaining the input-output relationship. It describes the technical relationship between inputs and output in physical terms.

5.0 Summary

The major issue of interest in this unit has been on the theory of production. We noted that the theory of production attempts to provide theoretical answers to these questions, through abstract models built under hypothetical conditions. Production function is a tool of analysis used in explaining the input-output relationship. It describes the technical relationship between inputs and output in physical terms. A production function is based on the following assumptions:

(i) perfect divisibility of both inputs and output;

(ii) there are only two factors of production – capital (K) and lacour (L);

(iii) limited substitution of one factor for the other;

(iv) a given technology; and,

(v) inelastic supply of fixed factors in the short-run.

Specific emphasis was on the Cobb-Douglas production function, using labour and capital as input combinations.

6.0 Self-Assessment Exercise

State and illustrate the Cobb-Douglas production function

7.0 References/Further Reading

Dwivedi, D.N. (2007). *Managerial Economics*. (6th ed.). Delhi: Gajendra Printing Press

Nicholson, W. (1978). *Microeconomic Theory: Basic Principles and Extensions.* (2nd ed.). Illinois: The Dryden Press

Unit 2 Optimal Input Combinations

1.0 Introduction

Economists are of the opinion that profit-maximising firms seek to minimise costs for a given level of output, or to maximise its output for a given total cost. The two major instruments in the maximisation of output are the ***Isoquants curves*** and ***Isocost line***, often referred to as the ***budget constraint line.*** The logic of isoquant tells you that a given level of output can be produced with different input combinations. Given the input prices, however, only one of the input combinations would be the least cost combination. The least-cost combination represents the input combination for which the budget constraint line is tangent to the isoquant curve. In this unit, we continue our discussions on production functions with the way in which optimal input combinations can be determined.

2.0 Objectives

At the end of this unit, you should be able to:

- know how to come up with the optimal input combinations
- understand the two major instruments in the maximisation of output, the isoquant curves and the budget constraint
- understand how changes in input prices can affect optimal input combinations
- make strategic input combinations.

3.0 Main Content

3.1 The Least-Cost Combination of Inputs

The optimal input combination is the point for which the slope of the budget constraint line equals the slope of the isoquant curve, as indicated by figure 3.1.1 below. It is also the least-cost combination point.

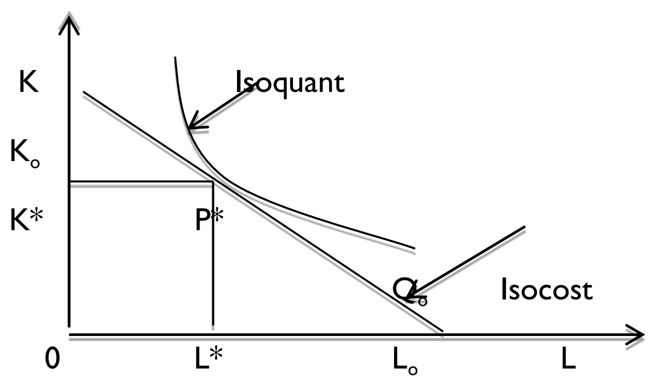


Figure 3.1.1: Least-Cost Combination of Inputs

According to figure 3.1.1, the least-cost combination of the inputs, capital (K) and labour (L) is at the point (P*) for which the isocost line, K_oL_o is tangent to the isoquant curve, Q_o. At this point, the optimal combination of capital (K) and labour (L) is OK* of K and OL* of L. This combination is optimal since it satisfies the least-cost criterion:

$$\frac{MP_L}{MP_K} = \frac{P_L}{P_K} \qquad (3.1.1)$$

$$\text{Or } \frac{MP_L}{P_L} = \frac{MP_K}{P_K} \qquad (3.1.2)$$

where MP_L and MP_K are marginal products of labour and capital, respectively, and P_L and P_K are prices of labour and capital, respectively.

The above least-cost criterion can be translated in values terms by multiplying the marginal productivities of capital (MP_K) and labour (MP_L) each by the product price (P) to obtain the marginal revenue product of labour (MRP_L) and the marginal revenue product of capital (MRP_K), and taking ratios to get:

$$\frac{MP_L.P}{MP_K.P} = \frac{MRP_L}{MRP_K} \qquad (3.1.3)$$

Equation (3.1.3) can be related to the ratio of input prices as follows:

$$\frac{P_L}{P_K} = \frac{MRP_L}{MRP_K}$$

$$\text{Or, } \frac{MRP_L}{P_L} = \frac{MRP_K}{P_K} \qquad (3.1.4)$$

It can be inferred from equation (3.1.4) that least-cost or optimum input combination requires that the marginal revenue productivity ratio of factors should be equal to their price ratios, or that the marginal revenue productivity and factor price ratios of all the inputs must be equal.

3.2 Effect of Change in Input Prices on the Optimal Combination of Inputs

Changes in input prices affect the optimal combination of inputs at different magnitudes, depending on the nature of input price change. If all input prices change in the same proportion, the relative prices of inputs (that is the slope of the budget constraint or isocost line) remain unaffected. But when input prices change at different rates in the same direction, or change at different rates in the opposite direction, or price of only one input changes while the prices of other inputs remain constant, the relative prices of the inputs will change. This change in relative input-prices changes both the input-combinations and the level of output. The change in input-combinations is as a result of the substitution effect of change in relative prices of inputs. A change in relative prices of inputs would imply that some inputs have become cheaper in relation to others. Cost-minimising firms attempt to substitute relatively cheaper inputs for the more expense ones. This refers to the ***substitution effect*** of relative input-price changes.

The effect of change in input prices on optimal input combinations is illustrated by figure 3.2.1 below.

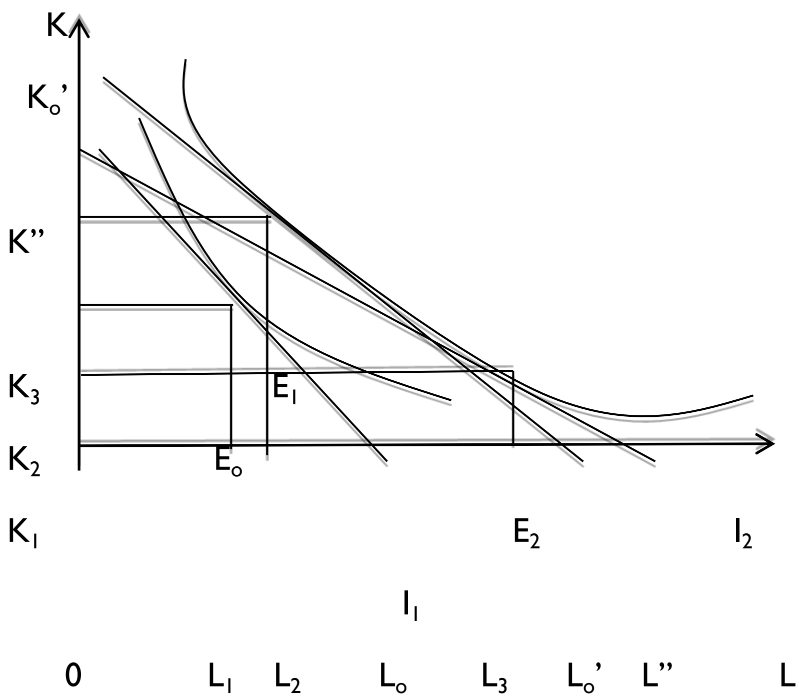


Figure 3.2.1: Substitution Effect of Changes in Input Prices

We assume that, given the price of capital (P_k) and price of labour (P_L), and the total resources as indicated by the isocost line, $K''L_o$, the representative firm's optimum input-combination is given by point E_o in figure 3.2.1. Suppose that P_L decreases (P_k remaining constant), resulting in a new isocost, K"L", which is tangent to the isoquant, I_2 at point E_2. At this point, the firm's new optimum combination of inputs becomes $OK_1 + OL_3$. It follows that the decrease in price of labour (P_L) has given rise to the reduction of capital input by the amount K_1K_2 and increment of labour input by L_1L_3. The change in the input combination is referred to as the ***price effect*** of the decrease in the price of labour. This price effect is composed of substitution and budget effects, where the substitution effect is represented by the difference between price effect and budget effect. Thus,

Substitution effect = Price effect – Budget effect.

From figure 2, the Price effect = L_1L_3, and,

Budget effect = L_1L_2

Substitution effect = $L_1L_3 - L_1L_2 = L_2L_3$

We conclude therefore, that a firm's input combination changes with a change in the price of a given input, all things being equal. In this illustration, the firm employs more of the cheaper input (L) and less of the more expensive one (K). The level of output also changes, as you can infer from figure 3.2.1

3.3 Theory of Cost and Break-Even Analysis

Business decisions are generally taken based on the monetary values of inputs and outputs. Note that the quantity of inputs multiplied by their respective unit prices will give the monetary value or the ***cost of production.*** Production cost is an important factor in all business decisions, especially those decisions concerning:

(a) the location of the weak points in production management;

(b) cost minimisation

(c) finding the optimal level of output;

(d) determination of price and dealers' margin; and,

(e) estimation of the costs of business operation.

In this section, we present briefly the cost concepts applicable to business decisions, cost-output relations, and the break-even analysis.

3.3.1 The Business Cost Concepts

The cost concepts are theoretically grouped under two over-lapping categories:

(i) Concepts used for accounting purposes; and,

(ii) Analytical cost concepts used in economic analysis of business activities.

Accounting Cost Concepts

The accounting cost concepts include:

1. Opportunity Cost and Actual or Explicit Cost. Opportunity cost can be seen as the expected returns from the second best use of an economic resource which is foregone due to the scarcity of the resources. Some scholars refer to opportunity cost as *alternative cost.* There would be no opportunity cost if the resources available to the society were unlimited.

Associated with the concept of opportunity cost is the concept of ***economic rent*** or ***economic profit.*** Economic rent is the excess of earning from investment over and above the expected profit. The business implication of this concept is that investing in a given project will be preferred so long as its economic rent is greater than zero or positive. Additionally, if firms know the economic rent of various alternative uses of their resources, it will aid them in the choice of the best investment avenue.

The actual or explicit costs are those out-of-pocket costs of labour, materials, machine, plant building and other factors of production.

Self-Assessment Exercise

Explain briefly what you understand by isoquant

4.0 Conclusion

The unit points out the fact that economists are of the opinion that profit-maximising firms seek to minimise costs for a given level of output, or to maximise its output for a given total cost. The two major instruments in the maximisation of output are the Isoquants curves and the budget constraint line. The optimal input combination is the point for which the slope of the budget constraint line equals the slope of the isoquant curve

Changes in input prices affect the optimal combination of inputs at different magnitudes, depending on the nature of input price change. If all input prices change in the same proportion, the relative prices of inputs (that is the slope of the budget constraint or isocost line) remain unaffected.

Business decisions are generally taken based on the monetary values of inputs and outputs, where the quantity of inputs multiplied by their respective unit prices will give the monetary value or the cost of production.

5.0 Summary

The optimal input combination is the point for which the slope of the budget constraint line equals the slope of the isoquant curve.

The logic of isoquant tells you that a given level of output can be produced with different input combinations. Given the input prices, however, only one of the input combinations would be the least cost combination. The least-cost combination represents the input combination for which the budget constraint line is tangent to the isoquant curve. In this unit, we continue our discussions on production functions with the way in which optimal input combinations can be determined.

Cost-minimising firms attempt to substitute relatively cheaper inputs for the more expense ones. This refers to the ***substitution effect*** of relative input-price changes.

Business decisions are generally taken based on the monetary values of inputs and outputs.

Production cost is an important factor in all business decisions, especially those decisions concerning: the location of the weak points in production management; cost minimisation; finding the optimal level of output; determination of price and dealers' margin; and, estimation of the costs of business operation.

6.0 Self-Assessment Exercise

1. What do you understand by optimum input combination?

2. What are the criteria for the least-cost combination of inputs?

7.0 References/Further Reading

Dwivedi, D.N. (2007). *Managerial Economics*. (6th ed.). Delhi: Gajendra Printing Press.

Nicholson, W. (1978). *Microeconomic Theory: Basic Principles and Extensions* (2nd ed.). Illinois: The Dryden Press.

Unit 3 The Cost Functions

1.0 Introduction

Economists often assume that the problem of optimum input combinations has been solved and attempts to conduct their analysis of the firm's behaviour in terms of revenues and costs expressed as functions of output. The problem of production is then to select an output level at which profits are maximised. In this unit, we examine the cost functions with aim of identifying the different categories of production costs and how they can be minimised.

2.0 Objectives

At the end of this unit, you should be able to:

- list the types of costs in a given production process
- define average and marginal costs
- explain the difference between Accounting and Economic profits
- explain the relationship between short-and long-run costs of a given firm.

3.0 Main Content

3.1 Definitions of Costs

Economists define three different concepts of costs: opportunity cost, accounting cost, and "***economic***" cost. For most economists, the most important of these is the *social* or *opportunity cost*. Due to limited resources, any decision to produce some goods necessitates some trade-offs or doing without some other goods. When some tubers of yams are produced, for example, an implicit decision has been made to do without, say 10 kilograms of beans that could have been produced using the available resources that went into the production of the tubers of yam. The opportunity cost of the tubers of yams is then 10 kilograms of beans.

The opportunity cost doctrine is extremely important in economic analysis. Many problems of social choice are often made conceptually clearer by recognising the alternatives inherent in a given economic process.

The two other concepts of cost are directly related to the firm's theory of choice. These include the accountant's concept of cost and the economist's concept of the firm's costs. The accountant's concept of costs stresses out-of-pocket expenses, historical costs, fixed costs, depreciation, and other book-keeping entries. The economists' definition uses the idea of opportunity cost, and defines cost to be that payment necessary to keep a resource in its present use or employment.

We can use a simple relationship to distinguish between ***accounting costs*** and ***economic costs***:

Accounting Costs = Total Variable Costs (TVCs) + Total Fixed Costs (TFCs)

Economic Costs = Accounting Costs + Opportunity Costs

3.2 Cost Functions

Cost functions are often classified into ***short-run*** and ***long-run*** functions. We summarise these in the following discussions.

3.2.1 Short-Run Cost Functions

We consider a system of equations consisting of:

(i) A production function of the form: Q = f(K, L) (3.1) where K and L represent capital inputs and labour inputs, respectively. The unit price of capital can be represented by the interest rate (r), and the unit price of labour by the wage rate (w).

(ii) A cost function: C = rK + wL (3.2)

Equations (3.1) and (3.2) can be reduced to a single equation in which cost is stated as an explicit function of the level of output (Q) and input prices (r and w), plus cost of fixed inputs (or fixed costs), b:

$$C = C(Q, r, w) + b \quad (3.3)$$

Equation (3.3) becomes the short-run cost function.

In the short run however, it is assumed that input prices are invariant (do not vary), so that the cost function may be stated simply as a function of the output level, Q, plus the fixed cost:

$$C = C(Q) + b \quad (3.4)$$

The cost of the fixed inputs, fixed cost, must be paid regardless of the quantity of output, or whether zero unit is produced. The cost function gives the minimum cost of producing each output and is derived on the assumption that the producer or entrepreneur acts rationally.

Number of special cost-output relations can be derived from equation (3.4), including:

(1) The Average total cost (ATC);

(2) The Average variable cost (AVC);

(3) The Average fixed cost (AFC); and,

(4) The Marginal cost (MC).

The Average Total Cost (ATC), Average Variable Cost (AVC), and the Average Fixed Cost (AFC) are defined as the respective total, variable, and fixed costs divided by the level of output:

$$ATC = \frac{C(Q) + b}{Q}$$

$$AVC = \frac{C(Q)}{Q}$$

$$AFC = b/Q$$

ATC is the sum of AVC and AFC:

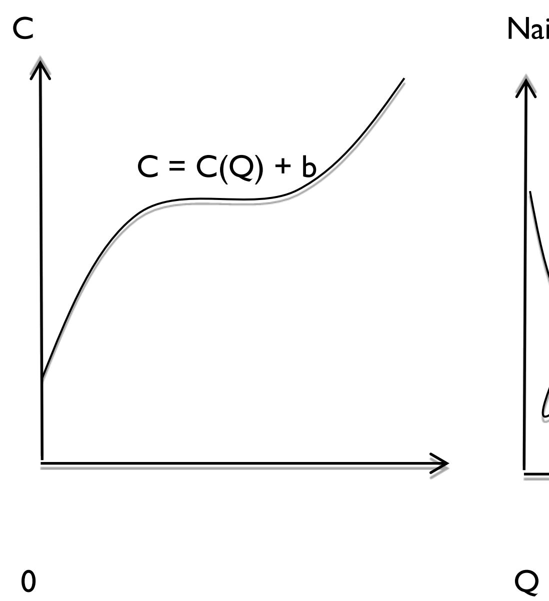
$$ATC = AVC + AFC \qquad (3.5)$$

The marginal cost, MC, is the derivative of the cost function with respect to output:

$$MC = dC/dQ = C'(Q).$$

Note that the derivatives of total cost (TC) and total variable cost with respect to output are identical since the term representing the fixed cost vanishes upon differentiation (derivative of a constant equals zero).

Specific cost functions assume different shapes. One possibility which exhibits properties often assumed by economists is shown in figures 3.1 and 3.2. Figure 3.1 indicates that total cost (C) is a cubic function of output (Q).



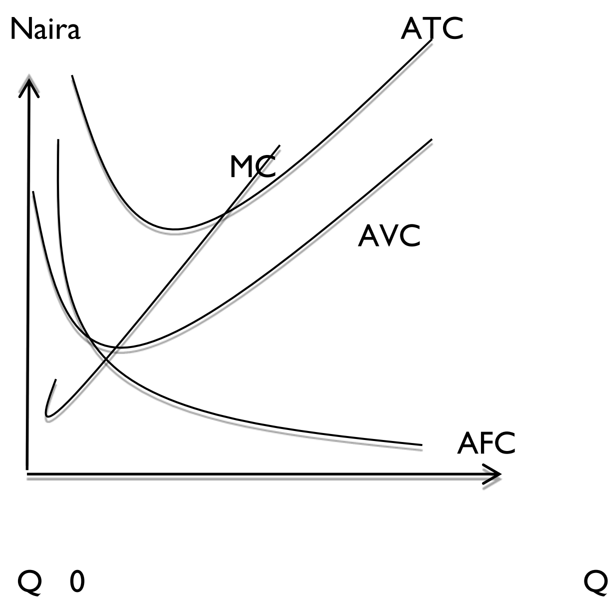


Figure 3.1: The Total Cost Curve **Figure 3.2: ATC, AVC, AFC, and MC Curves**

Figure 3.2 shows that ATC, AVC, and MC are all second-degree curves which first decline and then increase as output expands. MC reaches its minimum before ATC and AVC, and AVC reaches its minimum before ATC. The MC curve passes through the minimum points of both the AVC and ATC curves. The AFC curve is a rectangular hyperbola regardless of the shapes of the other cost curves; the fixed cost is spread over a larger number of units as output expands, thus, AFC declines monotonically. The vertical distance between the ATC and AVC curves equals the value of AFC, and hence decreases as output expands.

3.2.2 Long-Run Cost Functions

Let us assume that a given entrepreneur's fixed inputs are given by the parameter k, which represents the plant size. The greater the value of k, the greater the size of his plant. Let us also assume that the parameter k is continuously variable and we introduce it explicitly into the production function and the cost function:

$$Q = f(L,K,k) \qquad (3.6)$$

$$C = wL + rK + g(k) \qquad (3.7)$$

Fixed cost is an increasing function of the plant size: $g'(k) > 0$.

Eliminating the input prices, w and r, the total cost can be expressed as a function of output and plant size:

$$C = C(Q, k) + g(k) \qquad (3.8)$$

The equation (3.8) describes a family of short-run cost curves generated by assigning different values to the parameter k (different plant sizes). The equation represents the long-run cost function. The entrepreneurs long-run total cost function gives the minimum cost of producing each output level if he/she can vary his/her plant size. For a given level of output, he/she computes the total cost for each possible plant size and selects the plant size that minimises the total cost of production. Figure 3.3 presents the total cost curves corresponding to three different plant sizes.

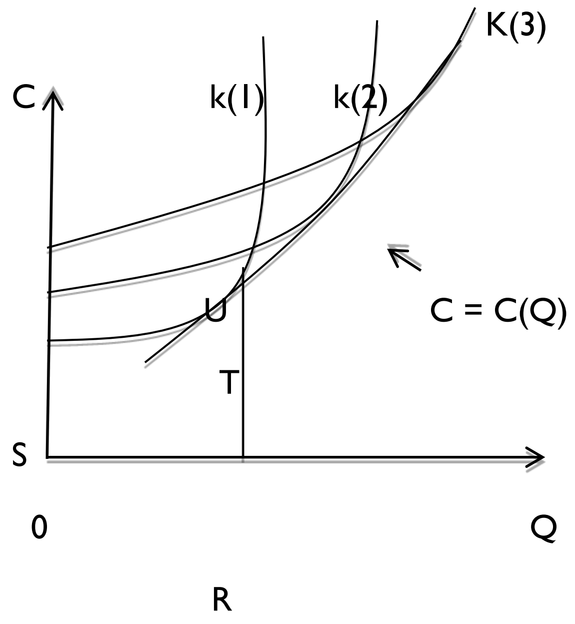


Figure 3.3: The Long-Run Cost Curve

According to figure 3.3, the entrepreneur can produce OR units in any of the plants, represented by k(1), k(2), and k(3). His/her total cost would be RS for plant size k(1), RT for k(2), and, RU for k(3). The plant size k(1) gives the minimum production cost for OR units of output. Therefore the point S lies on the long-run total cost curve, defined as the locus of minimum-cost points. The long-run cost curve is the envelope of the short-run cost curves, also represented in figure 3.3 as k(1), k(2), and k(3). The long-run cost function is shown in the figure as C = C(Q).

The long-run total cost curve is tangent to each short-run cost curve at the output level for which the short-run curve in question represents optimum plant size. The MCs can be defined as the slopes of the tangents of these short-run curves, and the long-run and short-run MCs are equal at such points.

Self-Assessment Exercise

Discuss the major differences between Accounting and Economic profits.

4.0 Conclusion

The unit began by explaining production costs in terms of two factors of production, capital and labour inputs. You were informed that production costs are generally classified into total, average, and marginal costs. You also learned that the major difference between accounting and economic profit is opportunity costs of production. We were also interested in the difference between short-and long-run costs. In sum, we observed that the long-run cost curve is the envelope of a series of short-run costs curves, as modeled by a firm's plant size.

5.0 Summary

Economists define three different concepts of costs: opportunity cost, accounting cost, and "economic" cost. For most economists, the most important of these is the *social* or *opportunity cost.*

We can use a simple relationship to distinguish between ***accounting costs*** and ***economic costs*** as follows:

Accounting Costs = Total Variable Costs (TVCs) + Total Fixed Costs (TFCs)

Economic Costs = Accounting Costs + Opportunity Costs

Cost functions are often classified into ***short-run*** and ***long-run*** functions.

A short-run cost function is basically made up of variable and fixed costs, as well as implicit or opportunity costs. In the short run, a firm is often interested in marginal costs of production, defined as the extra cost of producing additional units of output. The long-run average cost is the envelope of short-run average costs.

6.0 Self-Assessment Exercise

Consider the cubic total cost function: $0A\ .04Q^3 - 0.9Q^2 + 10Q + 5$

If the firm with this cost function produces 100 units of output:

(a) Compute the total cost of production

(b) Compute the average cost of production

(c) Compute the marginal cost of production.

7.0 References/Further Reading

Henderson, J. M. & Quandt, R. E. (1980). *Microeconomic Theory: A Mathematical Approach.* New York: McGraw-Hill Book Company.

Nicholson, W. (1978). *Microeconomic Theory: Basic Principles and Extensions* (2nd ed.). Illinois: The Dryden Press.

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Unit 4 Profit Maximisation by Linear Programming Methods

1.0 Introduction

For most business organisations, optimisation essentially implies profit and/or revenue maximisation or cost minimisation, given productive resources and input prices. The conventional theories of production give optimum solution in some abstract quantitative terms, through diagrams, mathematical derivations, logic, and symbols. They attempt to provide only guidance to decision-makers in their efforts to optimise resource allocation. They fail to provide exact solutions to practical problems of business. You should note that real life problems of maximisation and minimisation are much more complex than accounted for by the conventional theories of economics. These problems require real solutions in quantitative terms.

In the real business world, decision-makers deal with a large number of variables, with many constraints, in their efforts to arrive at an optimum solution to the problem of resource allocation. This problem is better solved by applying a sophisticated mathematical technique, referred to in this unit as *linear programming*. Linear programming is highly significant to business decisions, as it helps in measuring complex economic relationships and thereby provides an optimum solution to a resource allocation problem. Linear programming techniques thus, bridge the gap between abstract economic theories and managerial decision-making.

2.0 Objectives

At the end of this unit, you should be able to:

- define the basic concepts in linear programming techniques
- enumerate the basic assumptions of linear programming
- explain the linear programming technique in optimisation problems
- describe the efficient method of solving linear programming problems.

3.0 Main Content

3.1 Definition and Important Concepts in Linear Programming

Linear programming can be defined as a mathematical technique for solving maximisation and minimisation problems in business and economic decisions. It involves variables that have linear relationships with each other.

In linear programming, certain specific terminologies and concepts with specific connotations are used in the formulation of problems. You need to be familiar with these concepts, as discussed below.

3.1.1 Objective Function

In linear programming, objective functions are expressed in the form of equations. They refer to the quantity either to be maximised or minimised. As an example, if a business firm produces and sells outputs, X_1, X_2, and X_3, each yielding unit profit of 10, 4, and 7, respectively, the *objective function* or profit maximisation function can be expressed in the form:

Maximise □ = $10X_1 + 4X_2 + 7X_3$ (3.1.1)

3.1.2 Constrained Optimisation

The objective functions for maximisation or minimisation problems are subject to constraints that can prevent the solutions from being infinitely large or small. These constraints specify the limiting conditions which arise either out of limited resources or technological limitations. As an example, you often observe that the maximisation of output is constrained by the limited availability of inputs, such as number of machines, hours of work, raw materials, and the like.

Constraints are expressed either in the form of equalities or in the form of inequalities. Assume a business firm has only 100 machine hours (M), and 1000 man-hours (L) available for its production process, you can express the production constraints as:

M □ 100

L □ 1000 (3.1.2)

3.1.3 Choice Variables

Choice variables are those variables chosen to maximise or minimise the objective function, satisfying all the constraints. Each choice variable is an indicator of the level of physical activity or operation, such as producing a commodity, buying or selling of goods, transporting of goods, and the like. A choice variable may also represent price, which does not indicate a physical activity.

3.1.4 Non-Negativity Condition

The non-negativity condition of variables in linear programming is an important requirement. This is because the variables business firms deal with, such as labour inputs, materials, space, output, and machines, cannot be negative. These variables can either be equal to or greater than zero. Using the variable in equation (3.1.2), for example, the non-negativity conditions can be expressed as follows:

M □ 0; L □ 0

3.1.5 Feasible and Optimum Solutions

Feasible solutions are those that you can achieve with a given amount of resources or according to the resource constraints. Recall the 'budget line' in the analysis of consumer behaviour, or the 'isocosts' in production analysis. All possible combinations on or below the 'budget line' or 'isocosts' are the feasible solutions. Note that feasible solutions do not have to satisfy all the constraints. It is the solutions that satisfy all the constraints that are known as the *optimum solutions* or *optimum feasible solutions*.

3.2 Assumptions of Linear Programming

Solutions to an optimisation or a minimisation problem in linear programming are based on the following assumptions.

3.2.1 Linearity

It is assumed that there exists a linear relationship between output and inputs. This linear input-output relationship is an assumption as well as a condition in linear programming solutions. Stated alternatively, linear programming solutions are based on the assumption of constant returns to a factor in the short run. This linear relationship is represented by a straight-line equation. For example, consider an automobile manufacturer requiring 200 labour hours (L), 250 machine hours (M), and 0.8 tonnes of steel (S) to produce 100 automobiles. The input-output linear relationship can be expressed as:

$$200L + 250M + 0.8S = 100 \qquad (3.2.1)$$

3.2.2 Continuity

Continuity requires that all variables are quantifiable in numerical values. This is because it is only numerical values that can provide continuity in measurement. In optimum solutions, non-numerical or unquantifiable values are meaningless.

3.2.3 Independence and Additivity

It is assumed that the variables and their quantitative specifications are independent of other variables. In other words, given the constraints, the variables should be capable of being arbitrarily chosen. These variables should also satisfy the condition of additivity, that is, the quality of being added together. Non-additive values cannot be used in linear programming solutions.

3.2.4 Proportionality

Proportionality implies that the linear relationship between the variables should be proportional and should not change in the course of solution. For instance, if production of 1 unit of a commodity requires 2 units of an input, production of 10 units would require 20 units of the input, and so on.

3.2.5 Constant Price

This is the assumption that input and output prices must be constant in linear programming problems, irrespective of the quantities purchased or sold. The price involved must be purely competitive price in both factor and commodity markets.

3.3 Applications of the Linear Programming Technique

In this section, you will see a practical business application of the linear programming techniques. We use the case of profit maximisation, for simplicity.

The Profit Maximisation Problem

You assume that a business firm produces two commodities, X and Y, with two different inputs, Labour (L) and Capital (K). The total quantities of L and K available per unit of time are specified as L = 1600 labour hours; and K = 2000 units. In addition, assume that producing 1 unit of commodity X requires 4 units of labour (L) and 2 units of capital (K). One unit of commodity Y requires 2 units of L and 5 units of K. Profits per unit of commodities X and Y are estimated at N10 and N8, respectively.

This information can be summarised in table 3.3.1 below

Inputs	**Total Inputs Available Per unit of time**	**Input Requirement per unit of product**	
		X	**Y**
Labour (L)	1600	4	2
Capital (K)	2000	2	5

Table 3.3.1: Production and Input Requirements

Given these information, the firm's objective is to maximise it total profit (☐). The problem is to choose an output mix of X and Y that maximizes profit.

Transformation of the Problem into Linear Programming

To transform the problem at hand to linear programming format, you need to restate the conditions of the problem in programming language. Take note of the following steps, they will help you.

Step 1: Specification of the Objective Function. The firm's objective function can be expressed in the following form:

Maximise ☐ = $10X + 8Y$ (3.3.1)

where X and Y represent quantities of commodities X and Y. When you multiply these quantities by their unit prices (or profits as the case may be) you will obtain the total profit (☐) as indicated by equation (3.3.1). It will be the linear programming technique that you will use in determining the units of X and Y to produce in order that profit will be maximised.

Step 2: Specification of the relevant Constraint Inequalities. Using the information available in table 3.3.1, you will formulate the relevant constrain equations as:

The constraint inequality for input L may be specified thus, 4X + 2Y □ 1600 (3.3.2)

The constraint inequality for input K may similarly be specified as,

2X + 5Y □ 2000 (3.3.3)

Step 3: Specification of Non-negative Conditions. Note that a negative quantity in optimum solutions is not allowed and does not make economic sense so that, you must impose non-negative conditions in the linear programming problem. The relevant non-negative conditions for the problem at hand can be expressed as:

X □ 0 and Y □ 0 (3.3.4)

You are now in a position to formulate the required linear programming problem in terms of equations and inequalities. The *problem* becomes:

Maximise □ = 10X + 8Y (the *objective function*),

Subject to the constraints:

4X + 2Y □ 1600 (3.3.5)

2X + 5Y □ 2000, (3.3.6)

where X □ 0 and Y □ 0

You will obtain the optimum solution to the problem at hand by solving for the values X and Y in the above equations.

3.4 Methods of Solving Linear Programming Problems

You will now be introduced to two popular methods of solving linear programming problems:

(i) Graphical Method

(ii) Simplex Method

3.4.1 The Graphical Method

This method is the simplest technique in solving linear programming problem. You begin by converting the constraint inequalities into equalities, and them sketching them in a graph. Thus, the constraint inquality (3.3.5) becomes:

4X + 2Y = 1600 (3.4.1)

and that of (3.3.6) becomes:

2X + 5Y = 2000 (3.4.2)

Notice that equations (3.4.1) and (3.4.2) are linear equations in X and Y. To sketch these equations, you will begin by determining the intercept terms for the two-dimensional graph in X and Y. Thus, to graph equation (3.4.1), you obtain the Y- and X-axis as,

For the *Y-axis*: $4X + 2Y = 1600$

$$2Y = 1600 – 4X$$

When $X = 0$, $2Y = 1600$

$$Y = 800$$

The Y-intercept is therefore, 800.

For the *X-axis*: $4X + 2Y = 1600$

$$4X = 1600 – 2Y$$

When $Y = 0$, $4X = 1600$

$$X = 400$$

The X-intercept is therefore, 400.

Similarly for equation (3.4.2), you obtain the Y- and X-axis as:

$2X + 5Y = 2000$

$5Y = 2000 – 2X$

When $X = 0$, $5Y = 2000$

$$Y = 400$$

Here, the Y-intercept is 400, and,

$2X + 5Y = 2000$

$2X = 2000 – 5Y$

When $Y = 0$, $2X = 2000$

$$X = 1000$$

Therefore, the X-intercept is 1000.

The sketches are as in figure (3.4.1) below:

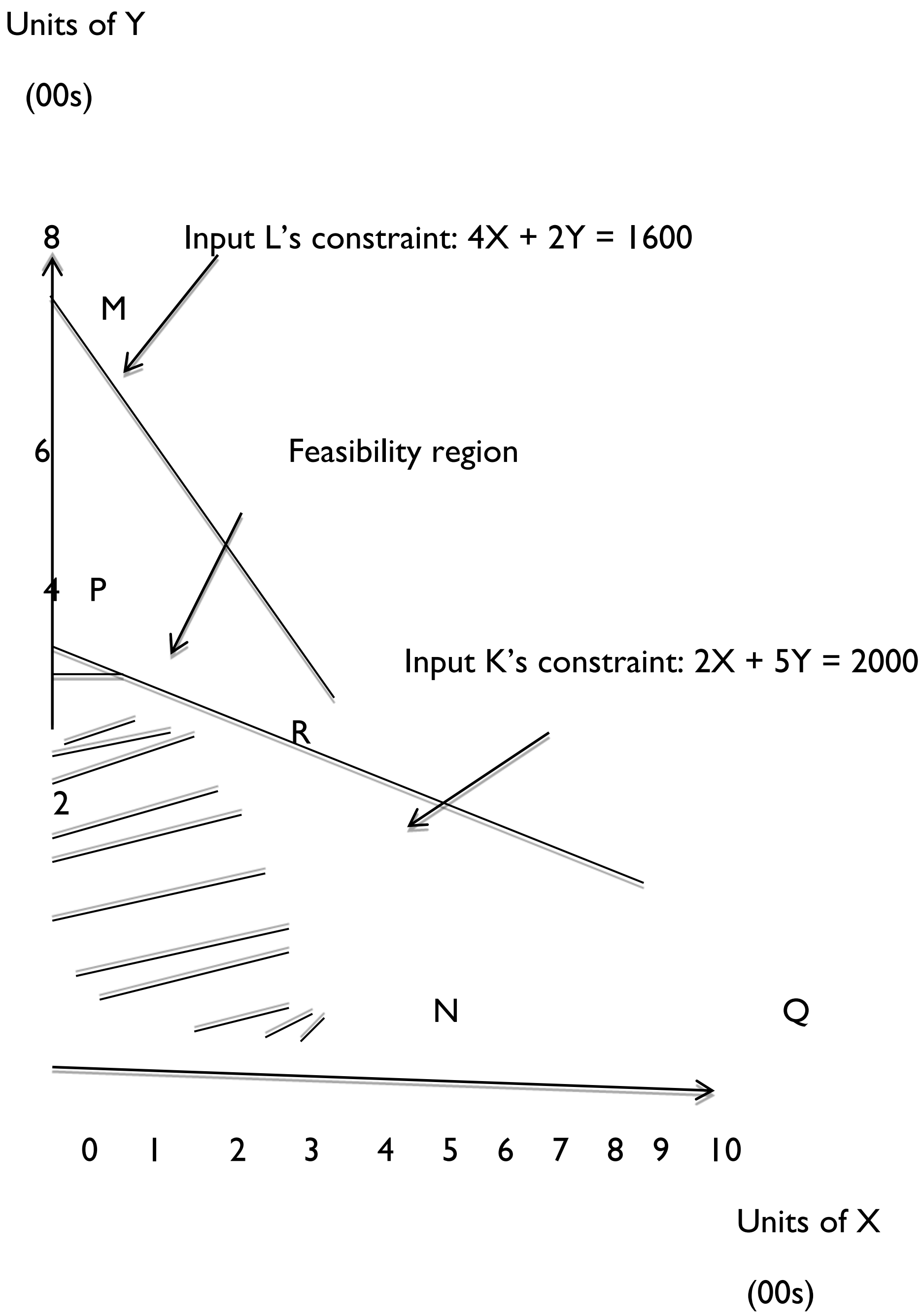

Figure 3.4.1: Production Constraints and Feasibility Region

Observe that in figure 3.4.1, the line MN is formed by joining the Y- and X-intercepts for the labour (L) constraint equation, and that of PQ is formed by joining Y and X-intercepts for the capital (K) constraint equation. All the points on the line MN satisfy the constraint, 4X + 2Y □ 1600. The area under OMN is referred to as the feasibility space for the single input, L. This implies that any point within the feasibility space and on the border lines is a feasible point for this input.

Similarly, the area under OPQ is referred to as the feasibility space for the single input, K. All the points on the line PQ satisfy the constraint,

2X + 5Y □ 2000.

The shaded area under OPRN represents the feasible region, where you will obtain the feasible output choices. Each of these choices satisfies both the constraints and the stated non-negativity conditions. Only those points falling under the feasible region satisfy all the feasibility conditions. Any point to the right of the area marked MRP represents a combination of the commodities X and Y that cannot be produced within the limited availability of the inputs L and K. All the points marked by the area PMR satisfy only the constraint, 4X + 2Y □ 1600. Similarly, all the points within the area marked NRQ satisfy only the constraint, 2X + 5Y □ 2000. It follows that only the feasible area, OPRN that meets the constraints and contains the point of solutions to the profit maximisation problem.

Your next step is to locate the point on the boundary of the feasible area. This point will represent the combination of the commodities X and Y that maximises profit. You can do this by graphing or sketching the objective function in the form of *isoprofit* lines, for different output levels, and superimposing these over the feasible region.

Graphing of the objective function would require finding the slope of the objective function, which you can do as follows:

Given the objective function, □ = 10X + 8Y, you write it in terms of Y to get:

Y = □/8 - 10/8X = □/8 – 1.25X

where □ = 0,

Y = -1.25X.

The coefficient, -1.25 gives the slope of the isoprofit line. This means that 1.25 units of commodity Y would yield the same profit as 1 unit of X. With this slope, a series of isoprofit lines can be drawn and superimposed over the feasibility region, as you can observe in figure 3.4.2 below. Note that since profitability of the two commodities is constant, isoprofit lines are parallel to each other.

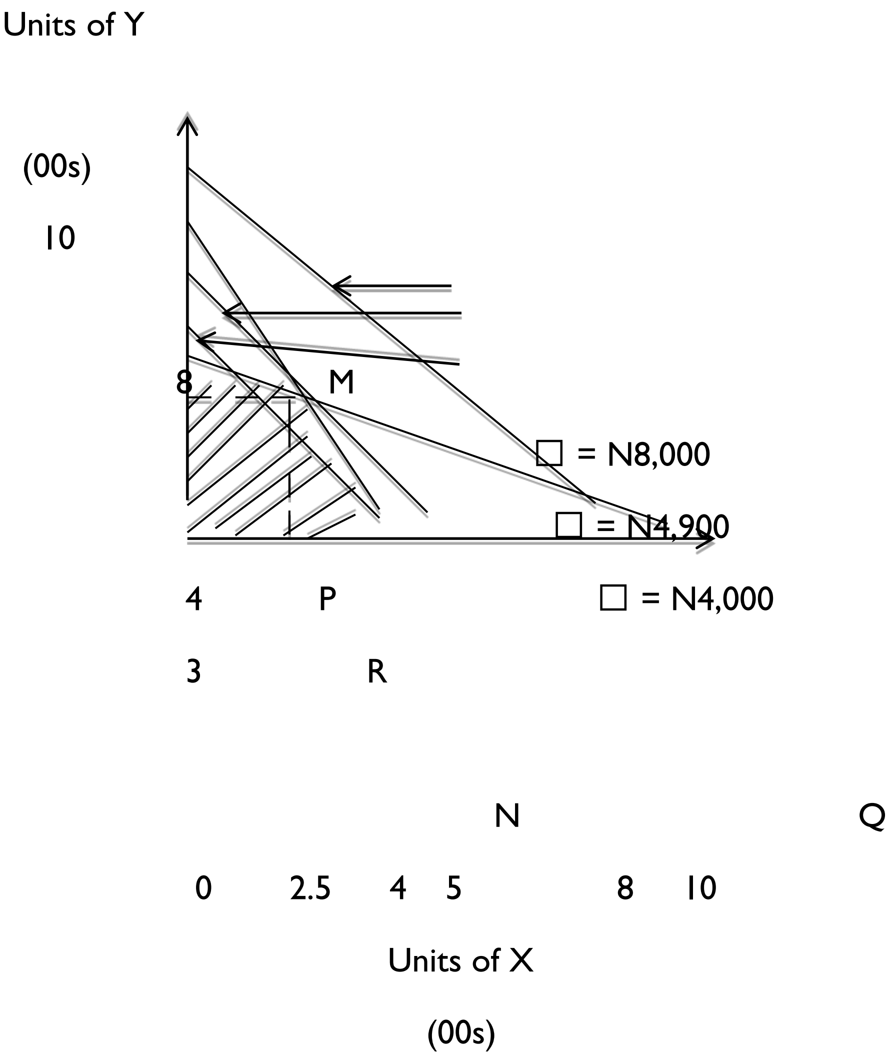

Figure 3.4.2: Graphical Solution of the Profit Maximisation Problem

Observe that the isoprofit line marked N8,000 is not possible because it lies very much above the feasibility space or region. The isoprofit line marked N4,000, passing under the feasibility space reflects an underutilisation of inputs. It therefore, indicates a less than maximum profit. The shaded area to the right of this line indicates the scope for increasing profit. The isoprofit line marked N4,900, which is tangent to the boundary of feasibility space at point R is the highest possible isoprofit line, representing the maximum possible profit given the resource constraints. The tangential point, R which represents a combination of X and Y (that is, 2.5(100) = 250 units, and 3(100) = 300 units), yields the maximum profit. Thus,

Maximum Profit (□*) = 10(250) + 8(300) = N4,900.

It follows that the profit maximising units of commodities X and Y are 250 units and 300 units, respectively.

3.4.2 The Simplex Method

Before you apply the simple x method, you need to introduce an additional variable, known as the slack variable. This slack variable is added to the constraints in the profit-maximising or cost minimising problems. The slack variable is meant to account for the amount of unused inputs at the point of solution. Slack variables are always non-negative since a negative slack variable would call for additional inputs, which will go against the production constraints.

By introducing slack variables, you will be converting inequalities of the constraint equations into equations. For example, after the introduction of the slack variables, S_a and S_b, the constraints represented by equations (3.3.5) and (3.3.6) are converted to the following equations:

$$4X + 2Y + S_a = 1600 \qquad (3.4.3)$$

$$2X + 5Y + S_b = 2000 \qquad (3.4.4)$$

where S_a □ 0 and S_b □ 0, and represent the unused quantities of the labour (L) and capital (K), respectively.

The introduction of slack variables also gives the following important information:

The maximum values of slack variables, S_a and S_b, are 1600 and 2000 respectively, when $X = 0$ and $Y = 0$.

If $S_a = S_b = 0$ at the optimum solutions, the inputs are fully used and excess capacity does not exist.

If $S_a > 0$ and $S_b > 0$, there exists excess capacity.

To solve equations (3.4.3) and (3.4.4), given the objective function in equation (3.3.1), you first determine algebraically the *corner* solutions. Consider the constraint equations first:

$$4X + 2Y + S_a = 1600$$

$$2X + 5Y + S_b = 2000$$

These equations contain four variables in all, these are X, Y, S_a, and S_b. You set the main variables, X and Y each equal to zero (conditions that exist at the corner points of origin). And when $X = 0$ and $Y = 0$, then from the constraint equations above, $S_a = 1600$ and $S_b = 2000$.

Substituting zero for X and zero for Y in the objective function, you will get:

$$Z = 10X + 8Y = 10(0) + 8(0) = 0$$

while $S_a = 1600$, and $S_b = 2000$ (as indicated above).

As indicated by the objective function, increasing the values or quantities of commodities X and Y will lead to increase in profit at the rate of 10 per unit of X, and 8 per unit of Y. Note that the values of X and Y can be increased to the limit determined by the constraints

Assume you decided to increase the value or quantity of Y. The limit to which you can increase Y is given by the constraint equations (3.4.3) and (3.4.4). The constraint equation (3.4.3), that is: $4X + 2Y + S_a = 1600$ allows you to increase the value of Y to the maximum of 800, when X = 0 and S_a = 0. Constraint (3.4.4), that is: 2X + 5Y + S_b = 2000, similarly allow you to increase the value of Y up to 400, when X = S_b = 0.

Observe that Y = 800 satisfies constraint (3.4.3) and not constraint (3.4.4), because 800(2) = 1600. But Y = 400 satisfies both constraint (3.4.3 and constraint (3.4.4), since 400(2) = 800 {which is within constraint (3.4.3)}, and 400(5) = 2000 {which is also within constraint (3.4.4)}. It follows that the upper limit of Y is 400 and not 800.

Y = 400 is an adjacent point to the origin (a corner point), when X = 0, and substituting 400 for Y and 0 for X in the objective function, you get:

Z = 10(0) + 8(400) = N3,200.

The objective is however, not to find the total profit at any of the corner points, but to know whether the N3200 obtained is the maximum profit. You have two ways of knowing whether N3200 is the maximum profit.

First, you can calculate the total profit at all corner points and compare them with the above profit, N3,200.

Second, you can determine algebraically whether the total profit can be increased beyond N3,200.

The simple x method uses the second approach. Since Y has been determined to be at a maximum permissible under the given constraints, there is no scope for increasing profit by increasing the production of Y. It follows that the only available opportunity for increasing profit is to make X > 0. But given the input constraints, you can only increase the production of commodity X by reducing the production of commodity Y. Therefore, the change in profit will depend on the relationship between commodity X and commodity Y, and also on the slack variables. You will be therefore required to express the profit or the objective function in terms of X and the slack variables. You will also be required to find the limit to which commodity X can be increased. The procedure for substituting X and S_b for Y in the profit function is exemplified as follows.

Using equation (3.4.4), you obtain:

$$5Y = 2000 – 2X - S_b$$

$$Y = 400 – 0.4X – 0.2S_b \qquad (3.4.5)$$

Substitute equation (3.4.5) for Y in the profit or objective function, you get:

$$Z = 10X + 8(400 – 0.4X – 0.2S_b)$$

$$= 10X + 3200 – 3.2X – 1.6S_b$$

$$= 3200 + 6.8X – 1.6S_b \qquad (3.4.6)$$

Equation (3.4.6) reveals the following important information:

At the point where X = S_b = 0, Profit (☐) = N3,200

Increasing X will increase profit, since $X > 0$ (positive coefficient)

Increasing S_b will decrease profit, since $S_b < 0$ (negative coefficient)

This information reveals the main logic of the simplex method.

With this logic, you can now proceed to solve the profit maximisation problem, using the simplex method.

As noted, increasing X will increase the total profit, but up to the limit determined by the constraints. The constraint, $4X + 2Y + S_a = 1600$ indicates that X can be increased up to the limit of 400 units, when $Y = S_a = 0$. The constraint,

$2X + 5Y + S_b = 2000$ indicates that X can be increased up to the limit of 1000 units, when $Y = S_b = 0$. While the later satisfies only one constraint, equation (3.4.3), the former satisfies all the constraints, including equation (3.4.4). Thus, 400 is the upper limit of X when Y, S_a □ 0. But since $S_b > 0$, the profit is not maximum because some units of the input, K, remain unused.

You are therefore, required to compute the profit at the point where $S_a = S_b = 0$. The profit function should therefore, be expressed in terms of S_a and S_b and equation (3.4.3) to be solved for X in terms of S_a and S_b as follows:

Substitute equation (3.4.4) for Y in equation (3.4.3), that is, $4X + 2Y + S_a = 1600$, you get:

$$4X + 2(400 – 0.4X – 0.25S_b) + S_a = 1600$$

$$3.2X = 800 + 0.4S_b – S_a$$

$$X = 250 + 0.125S_b – 0.312\ S_a \qquad (3.4.7)$$

Substitute equation (3.4.7) for X in the profit or objective function (3.4.6), you will get:

$$Z = (\square) = 3200 + 6.8(250 + 0.125S_b – 0.312S_a) – 1.6S_b$$

$$= 3200 + 1700 – 0.75S_b – 2.12S_a$$

$$= 4900 – 0.75S_b – 2.12S_a. \qquad (3.4.8)$$

Equation (3.4.8) gives you the total profit at the corner points $S_a = S_b = 0$. For you to further increase profit, S_a or S_b must be increased. But in equation (3.4.8) the coefficients of S_a and S_b indicate that they are negative variables. Therefore, making $S_a > 0$ or $S_b > 0$ (that is increasing them) will actually reduce the profit. The maximum profit is therefore, N4,900, when $S_a = S_b = 0$.

By inspection, equation 3.4.7 implies that when $S_a = 0$ and $S_b = 0$, the value of

X = 250 units. Substitute X = 250 and maximum profit (□) = 4900 into the objective or profit function, you get:

$$4900 = 10(250) + 8Y$$

2500 + 8Y = 4900

8Y = 2400

Y = 300.

You can now conclude that the profit maximising levels of output are 250 units of commodity X and 300 units of commodity Y. This gives you the final solution to the linear programming problem in this discussion.

Self-Assessment Exercise

Enumerate and discuss the basic assumptions of linear programming

4.0 Conclusion

This unit has made you to know that optimisation basically implies profit and/or revenue maximisation or cost minimisation. When optimisation problems become complex, you should result to linear programming technique as the most efficient technique in solving the problems.

The unit has also exposed you to the necessary assumptions of linear programming, including: Linearity, continuity, independence and additivity, proportionality, and constant price. You learned the two important methods of solving linear programming problems: (i) the Graphical method, and, (ii) the Simplex method.

To put these methods in practice, the unit solves an optimisation problem involving profit maximisation, where extension discussions on objective functions, constraints, and slack variables were presented. You also were meant to understand that the most feasible method of solving linear programming problems is the simplex method.

5.0 Summary

Linear programming can be viewed as a mathematical technique for solving maximisation and minimisation problems in business and economic decisions. It involves variables that have linear relationships with each other.

There are some important concepts of linear programming. These include in a nutshell: the objective function; constrained optimisation; choice variables; non-negativity conditions; feasibility, and optimum solutions. The guiding assumptions of linear programming are: linearity, continuity, independence and additivity, proportionality, and constant price.

It is necessary to transform optimisation problems into the corresponding linear programming language before its applications. The following basic steps in this transformation need to be followed:

Step 1: Specification of the objective function

Step 2: Specification of the constraint inequalities

Step 3: Specification of the non-negative conditions.

Having transformed the optimisation problem to its linear programming equivalence, there exist two methods of solving it: (i) the Graphical method; and, (ii) the Simplex method.

Linear programming techniques remain the most efficient technique for solving complex real life optimisation problems in business. The current rapid increases in the use of computer technology may have, however, introduced faster and more accurate method of solving optimisation problems than linear programming techniques.

6.0 Self-Assessment Exercise

Discuss the various ways in which linear programming might be used in helping business decisions.

A manufacturer produces two products, X and Y in two steps on machines A and B. The processing times for the two products on the two machines are given in the following table:

Product	**Machine A**	**Machine B**
X	4 hours	5 hours
Y	5 hours	2 hours

Machine A has 8 hours available and Machine B has 120 hours available. Product X has profit of N10 per 100 units, and product Y has profit of N5 per 10 units. There is no restriction on sales. Formulate the linear programming problem and use the simplex method to find out the number of units of products X and Y that must be produced for the manufacturer to maximise profit.

7.0 References/Further Reading

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